Abstract of the research report

Functionals for signal and image reconstruction: properties of their minimizers and applications

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This manuscript describes the main results of my research since the end of my PhD until now. The context of my work is the reconstruction of signals and images by minimizing a cost-function composed of a data-fidelity term and a regularization term that conveys a prior model for the solution. Classically, these cost-functions are conceived either in the framework of Bayesian statistics or in a variational framework. One can nevertheless observe a gap between the models and the solutions, which is misleading in the applications. I have then formulated and studied the following problem: analyze the properties (the shape) of the minimizers as a function of the shape of the cost-function itself. The ultimate goal of this approach is to learn how to determine cost-functions in such a way that their minimizers fit the models.

In particular I studied the roles of the (non)smoothness and the (non)convexity of the cost-function. I have shown that if the regularization is non-smooth then the minimizers are constant on some regions (*stair-casing*), while if it is non-convex, the minimizers involve sharp edges (*edge-enhancement*). I introduced non-smooth data-fidelity terms in regularized cost-functions and showed that the minimizers fit exactly part of the data entries. Using this property, I proposed methods for detection and smoothing of outliers and impulse noise using ℓ_1 data-fidelity terms.

I have also considered problems related to the numerical calculation of the minimizers, including the cases of non-smooth or non-convex regularization terms.