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Estimation of binary images by minimizing convex criteria

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Abstract

Reconstructing binary images is a problem which arises in various application fields. In our paper, this problem is considered in a regularization framework: the sought solution minimizes an objective function (a criterion). Criteria defined over the set of the binary images are nonconvex and there are no general methods permitting to calculate the global minimum, while approximate solutions are very often of limited interest. On the other hand, general-purpose reconstruction techniques, based on convex criteria, yield continuous-valued smooth estimates which are far from binarity.

In this paper, we propose two methods which are based on convex criteria and introduce binarity only partially. More precisely, we construct objective functions whose minimizers are continuous-valued but have a quasi-binary shape. In other words, these estimates are composed essentially of binary-valued pixels while nonbinary pixels are rare. According to our approach, the construction of these objective functions is based on analytical considerations. The resultant methods are stable and numerically attractive.

1 Problem formulation

In various application fields, an unknown *binary image* $\boldsymbol{x}$ (its pixels are either 0 or 1) has to be recovered from noisy data $\boldsymbol{y} = \mathcal{A}\boldsymbol{x} + \boldsymbol{n}$ obtained at the output of a linear observation system $\mathcal{A}$, while $\boldsymbol{n}$ accounts for uncertainties. In this work, we focus on well-determined observation systems—where $\mathcal{A}$ is *well-conditioned.* Typical situations are character and text denoising (then $\mathcal{A} = I$ is identity), restauration of binary images degraded by channel noise in communications, *etc.* A popular approach to solve such an inverse problem is regularization [1, 8, 3, 6]: estimate $\widehat{\boldsymbol{x}}$ is the minimizer of an objective function $\mathcal{F}$, which combines fidelity to data and closeness to priors, expressed through regularizer Φ:

$$\widehat{\boldsymbol{x}} \quad := \quad \arg\min_{\boldsymbol{x}} \mathcal{F}(\boldsymbol{x}) \qquad (1)$$

$$\mathcal{F}(\boldsymbol{x}) \quad = \quad \|\mathcal{A}\boldsymbol{x} - \boldsymbol{y}\|^2 + \Phi(\boldsymbol{x}). \qquad (2)$$

In our context, Φ should account for both, the *binarity* of the image and its *correlated structure.*

2 Usual approaches

2.1 Binary Markov models

A natural way to model the sought image is to define Φ as the energy of a binary Markov random field:

$$\Phi(\boldsymbol{x}) \quad = \quad \sum_i \alpha_i x_i + \sum_{i\sim j} \beta_{i,j} x_i\, x_j \qquad (3)$$
$$\text{subject to} \quad x_i \in \{0,1\} \text{ for any } i \in \mathcal{S}$$

where $\mathcal{S}$ is the lattice of the sites of the image, $i \sim j$ means that i and j are neighbours, while $\beta_{i,j}$ and α_i are parameters. Under this form can be written the classical Ising model, the auto-logistic model, the spin-glass *etc.* [1, 4, 6]. An estimate (2-1), involving a prior (3), is conceptually satisfying since it accounts properly for the both, binarity and correlations.

Direct calculation of $\widehat{\boldsymbol{x}}$ is computationally prohibitive since it needs to compare $2^{\#\{\mathcal{S}\}}$ possible images ($\#$ denotes cardinality). Exact calculation of $\widehat{\boldsymbol{x}}$ is possible only for a special form of $\mathcal{F}$ [7] but it is cumbersome to be used in practice. In general, $\widehat{\boldsymbol{x}}$ cannot be found exactly. This estimate can be approximated using simulated annealing [6, 5], which calculation is quite costly. The *Iterated conditional modes* (ICM) algorithm [1] provides a local minimum of $\mathcal{F}$ which is often a very poor estimate.

2.2 Surrogate methods

These numerical intricacies discourage practical use of binary priors. Instead, general image reconstruction techniques, based on convex criteria, are preferred. However, the resultant solutions are far from being binary. In many situations, subsequent thresholding cannot provide a meaningful (binary) estimate.

Generalized Gaussian energies [2] constitute an important class of efficient convex regularizers:

$$\Phi(\boldsymbol{x}) = \sum_{i\sim j} \beta_{i,j} |x_i - x_j|^p \quad \text{where} \quad 1 < p \leq 2 \qquad (4)$$

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The case $p = 2$ is the popular quadratic regularization which is well-known to "oversmooth" abrupt transitions. Regularizers corresponding to $p \gtrapprox 1$ are better adapted to binary images since they favour local smoothing while preserving large transitions [2].

3 Quasi-binary estimates

The set of the binary images being discrete (hence nonconvex) any criterion defined over this set is nonconvex. Instead, we explore the possibility to define continuous-valued estimates, which minimize convex criteria but which estimates have a "quite binary" shape. These minimizers should be composed *essentially* of binary-valued pixels while nonbinary pixels should be rare.

3.1 Discouraging nonbinary values

Since the sought image is binary, it must satisfy $\boldsymbol{x} \in [0,1]^{\#\{\mathcal{S}\}}$. The latter constraint is convex and easy to implement. On the contrary, no convex constraint defends the recovery of pixels inside $]0,1[$. Subtracting $(x_i - \frac{1}{2})^2$ from $\mathcal{F}$ will only inhibit such a recovery, while it does not introduce correlations throughout the image. We hence formulate a criterion of the form:

$$\mathcal{F}(\boldsymbol{x}) = \|\mathcal{A}\boldsymbol{x} - \boldsymbol{y}\|^2 - \alpha \sum_i (x_i - \frac{1}{2})^2 + \Psi(\boldsymbol{x}) \quad (5)$$
$$\text{subject to } x_i \in [0,1] \text{ for any } i \in \mathcal{S}$$

where Ψ will account for the correlations and will be chosen convex. The second term in (5) is concave. But $\mathcal{A}$ is well-conditioned, so $\mathcal{F}$ will be convex if

$$\alpha \lessapprox \lambda_{min}, \quad \text{where } \lambda_{min} = \min\{\text{eig}\,(\mathcal{A}^T\mathcal{A})\} \quad (6)$$

Thanks to both, the constraint and the concave term, pixels of $\widehat{\boldsymbol{x}}$ are "pushed" to take a value on the boundary of $[0,1]$, namely 0 or 1.

3.2 Correlations

We consider two different choices for Ψ in (5).

Quadratic regularization An usual, pertinent choice is to take

$$\Psi(\boldsymbol{x}) = \sum_{i \sim j} \beta_{i,j}(x_i - x_j)^2 \quad (7)$$

The resultant $\mathcal{F}$ is a *quadratic criterion* constrained on the convex set $[0,1]^{\#\{\mathcal{S}\}}$. Fast algorithms for the calculation of $\widehat{\boldsymbol{x}}$ can then be implemented [10].

Nonsmooth regularization Recently we have established that regularization using *nonsmooth* potential functions yields estimates containing large regions where the differences between neighbouring pixels are null [9]. This is a precious property which suggest that the introduction of a modulus potential function $|.|$ in Ψ will generate estimates which are constant over large zones. So we take

$$\Psi(\boldsymbol{x}) = \sum_{i \sim j} \beta_{i,j}|x_i - x_j| \quad (8)$$

Recall that (5) favours (up to some degree) the recovery of binary pixels. This trend, joined to the trend of (8) to yield locally constant regions, will result in the reconstruction of large 0-valued and 1-valued regions in $\widehat{\boldsymbol{x}}$. Thus (8) *enforces* the binarity constraint.

Notice that the recent advances in nonsmooth convex optimization [11] provide efficient algorithms for the calculation of $\widehat{\boldsymbol{x}}$.

4 Illustrations

The examples below concern the denoising of an image, since $\mathcal{A} = I$ is identity (in this case, $\lambda_{min} = 1$). Regularizer Ψ is defined using the 8 nearest neighbours, while $\beta_{i,j} = \beta$ for any $i, j \in \mathcal{S}$.

4.1 Denoising of a phantom

The original phantom is given in Fig.1(left); it is composed of large constant (0-1) pathces. Data, presented in Fig. 1(right), are corrupted by white Gaussian noise with variance 1. Neither thresholding with respect to 1/2, Fig.2(left), nor ICM, Fig.2(right), provide a meaningful estimate. Quadratic regularization yields a very smooth estimate (Fig.3). A better denoising is obtained using a generalized Gaussian model (4) with $p = 1.2$—see Fig.4. Since $\mathcal{A} = I$, median filtering can be applied as well; the obtained solution (Fig.5) is comparable with Fig.4.

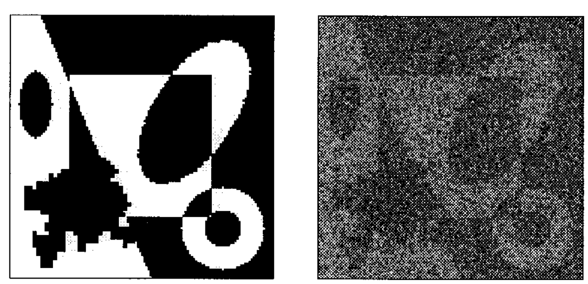

Fig. 1 Original phantom (left). Data corrupted by white Gaussian noise (right).

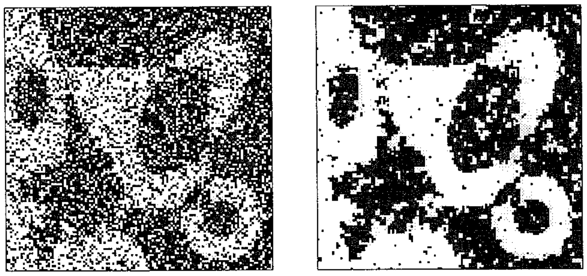

Fig. 2 Solutions obtained by direct thresholding (left) and by ICM (right).

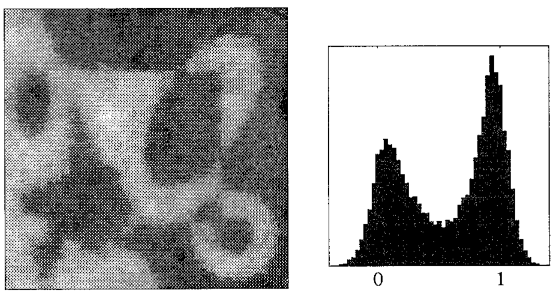

Fig. 3 Quadratic regularization (left). Histogram of the solution (right).

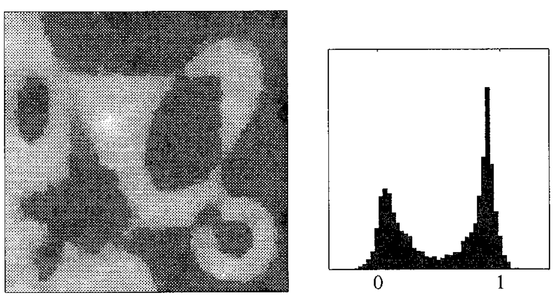

Fig. 4 Generalized Gaussian model.

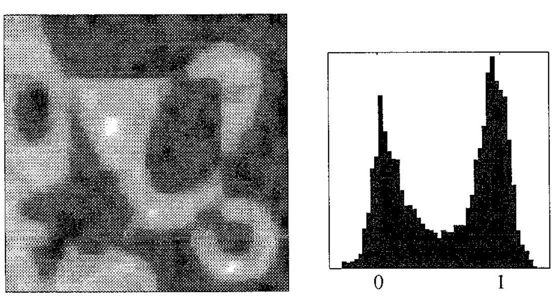

Fig. 5 Solution obtained using median filtering.

The first of the proposed methods (5,7), where Ψ is quadratic, permits to obtain a solution having only a small number of nonbinary pixels—see Fig.6. The second one—which involves nonsmooth regularization (8) in (5)—leads to an almost binary solution (Fig.7).

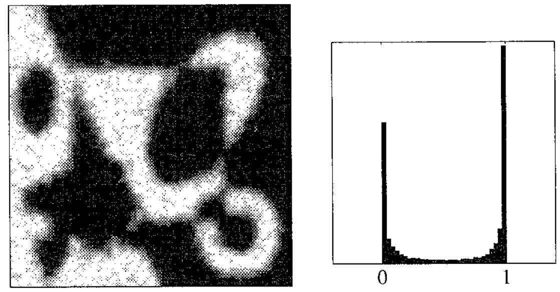

Fig. 6 First proposed method involving a quadratic regularization term (5,7).

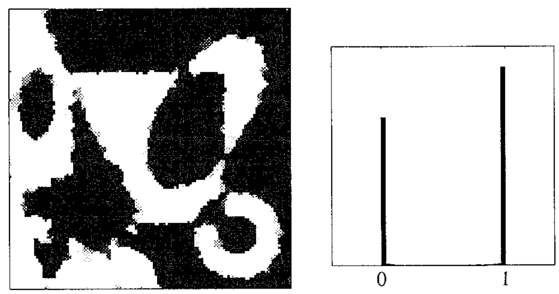

Fig. 7 Second proposed method involving nonsmooth regularization (5,8).

Data in Fig.8(left) are contaminated by a "salt and pepper" noise. These data are binary, hence bounded on $[0,1]^{\#\{S\}}$, so regularized solutions belong to $[0,1]^{\#\{S\}}$ as well. ICM does not permit to obtain a pertinent solution, as seen in Fig.8(right). The estimate obtained using quadratic regularization involves an important oversmoothing (Fig.9). Generalized Gaussian model ($p = 1.2$) lead to an estimate with proper homogeneous zones (Fig.10), although these are highly underestimated. In the context of "salt and pepper" noise, median filtering yields a solution where many pixels are binary (Fig.11).

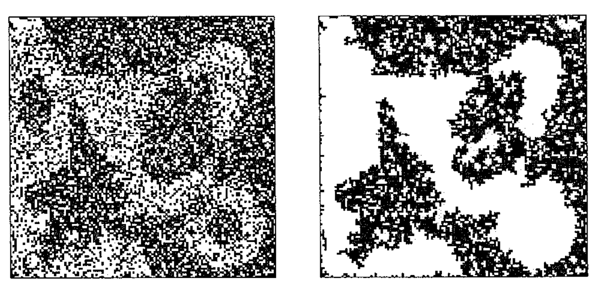

Fig. 8 Data degraded by a "salt and pepper" noise (left). Solution obtained by ICM (right).

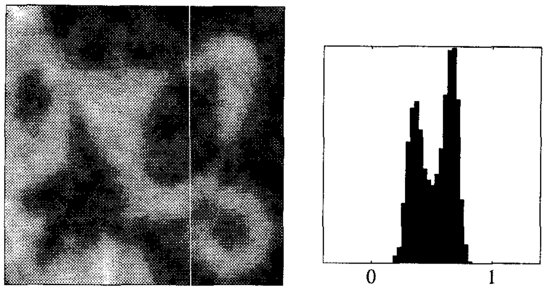

Fig. 9 Quadratic regularization.

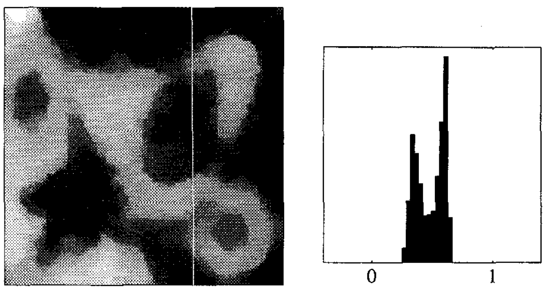

Fig. 10 Generalized Gaussian model.

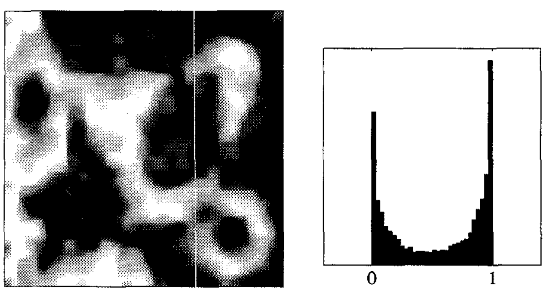

Fig. 11 Median filtering.

The first proposed method (5, 7) permits to find a better solution containing a few nonbinary pixels (Fig.12). The second method (5, 8) is more efficient again and yields a nice reconstruction (Fig.13).

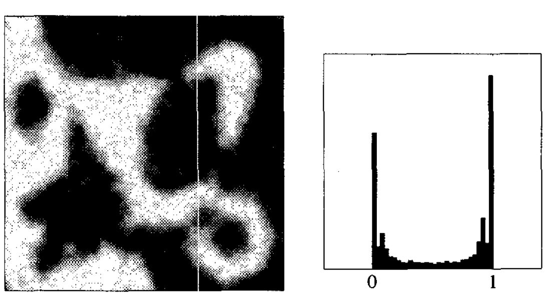

Fig. 12 First proposed method (with Ψ quadratic).

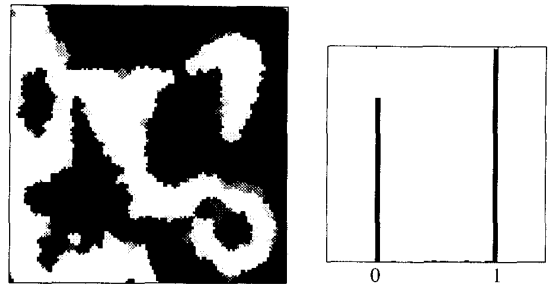

Fig. 13 Second proposed method (with Ψ nonsmooth).

4.2 Denoising of a word

The word to reconstruct is given in Fig.14(left). Data in Fig.14(right) contain white Gaussian noise with variance 0.8. The results obtained using the two proposed methods are presented in Figs.15 and 16.

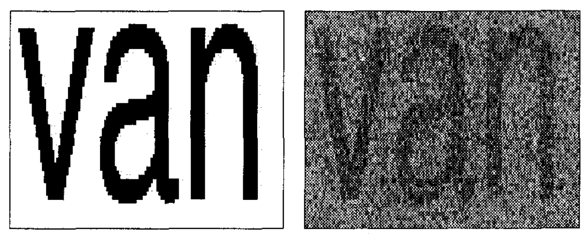

Fig. 14 Original word (left). Data degraded by white Gaussian noise (right).

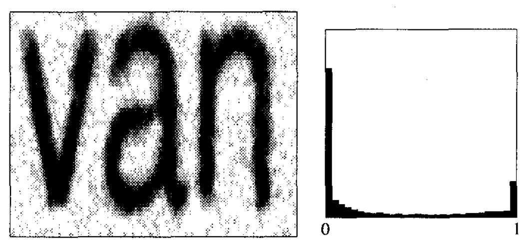

Fig. 15 First proposed method (Ψ is quadratic).

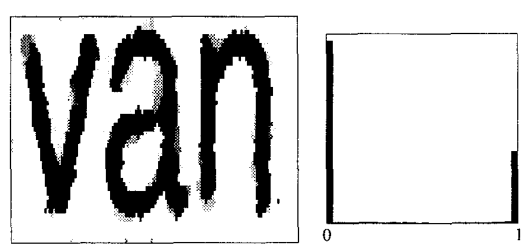

Fig. 16 Second proposed method (Ψ is nonsmooth).

A second set of data, Fig.17, contain binary "salt-and-pepper" noise. The two proposed methods, (5,7) and (5,8), yield the solutions given in Fig.18 and in

Fig.19, respectively.

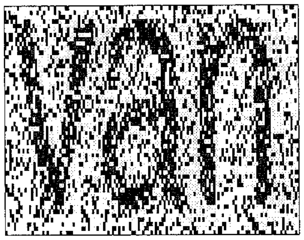

Fig. 17 Data degraded by "salt and pepper" noise.

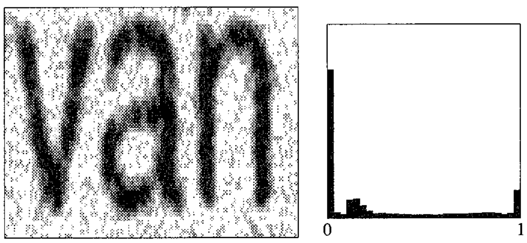

Fig. 18 First proposed method (Ψ is quadratic).

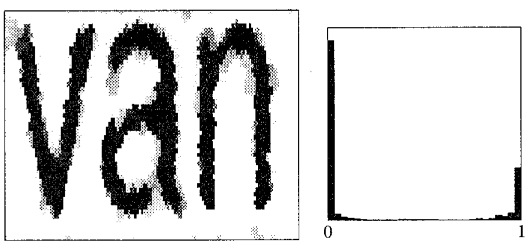

Fig. 19 Second proposed method (Ψ is nonsmooth).

In these reconstrutions, Ψ is defined over the 8 nearest neighbours again. However, it is seen in Fig.14(left) that the constant regions, corresponding to characters, are very tiny and have an elongated smooth shape. This fact suggests that special neighbourhood systems should be constructed, in order to express the main features of a character. We expect that the use of such properly adapted neighbourhoods will permit to improve the quality of the reconstructions obtainable using the proposed methods.

5 Conclusions

In this work, we propose a new approach for the reconstruction of binary images. Its main ambition is to conceive stable and numerically attractive methods, based on the use of convex criteria. Although strictly binary images cannot be estimated any more, our criteria yield quasi-binary continuous-valued images. The resultant methods allow the development of efficient implementations based on recent advances in smooth and nonsmooth optimization.

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