

# OPTIMIZATION THEORY - MAT4230 - CUHK, 2008

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## I. INTRODUCTION

- 1) Examples of optimization problems
  - 2) Convex sets and functions
  - 3) Existence, uniqueness
  - 4) Iterative methods, speed of convergence
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## II. UNCONSTRAINED PROBLEMS

- 1) Background
    - Existence
    - Optimality conditions
  - 2) First-order methods
    - Gauss-Seidel
    - Gradient methods
    - General descent schemes
    - Line-searches
  - 3) Second-order methods
    - Newton method
    - Quasi-Newton
    - DFP and BFGS
  - 4) Conjugate gradient (CG)
    - Basic method
    - Convergence
    - Application to non-quadratic functions
    - Relation with Quasi-Newton
  - 5) Special methods
    - Trust-regions
    - Gauss-Newton for Least-squares problems
    - Generalized Weiszfeld
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## III. CONSTRAINT OPTIMIZATION

- 1) Generalities
  - Optimality conditions
  - Projection theorem
  - Gauss-Seidel for a hyper-cube constraint
  - Gradient with projection
  - Penalty
- 2) Equality constraints
  - Lagrange Multipliers
  - Solving linear systems with constraints

## 3) Inequality constraints

- Abstract optimality conditions
  - Farkas-Minkowski lemma
  - Constraint qualification
  - Kuhn and Tucker conditions
  - The convex case
  - Dual problem
  - Uzawa method
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## IV. SELECTED TOPICS

- 1) Primal-dual methods
    - Convex conjugacy
    - Dual problems
    - Examples
  - 2) Introduction to nonsmooth optimization
    - Functions on  $R$
    - Convex functions on  $R^n$
    - Subdifferentials
    - Rademacher theorem
    - Optimality condition
    - Difficulties and descent directions
    - Subgradient method
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