Proc. IEEE Int. Conf. Acoust. Speech Signal Process., Detroit, Apr. 1995, pp. 1804-1807

PARAMETER SELECTION FOR A MARKOVIAN SIGNAL RECONSTRUCTION WITH EDGE DETECTION

Mila Nikolova

Laboratoire des Signaux et Systèmes (CNRS–Supélec–UPS)
Plateau de Moulon, 91192 Gif-sur-Yvette Cedex, France

ABSTRACT

We propose a method for the parameter selection for a Bayesian reconstruction of 1D or 2D signals, constituted by locally homogeneous regions, from incomplete and noisy projection data. A piecewise Gaussian Markov model (PG MM), defined as a sum of truncated quadratic potential functions, is used to regularise the reconstruction, which is otherwise ill-posed. This model is called the weak string in 1D and the weak membrane in 2D [1].

The posterior energy is highly non-convex and the MAP estimator is piecewise continuous; the model parameters play a particularly decisive role.

The resolution of the reconstruction – the finest recoverable features – is determined jointly by the parameters and the observation model. On the other hand, we propose a method for the determination of the parameters in order to reach, or at least to approach as closely as possible, a desired resolution. This model needs the evaluation of several posterior edge detection thresholds.

1. Introduction

We treat the general discrete linear observation model relating the original signal $\boldsymbol{x}^* \in \mathbb{R}^M$ to the noisy data $\boldsymbol{y} \in \mathbb{R}^N$ by

$$\boldsymbol{y} = \mathcal{A}\boldsymbol{x}^* + \boldsymbol{n}. \tag{1}$$

$\boldsymbol{x}^*$ is defined on an M-points lattice, say $\mathcal{S}$, which can be 1D or 2D. The observation operator $\mathcal{A}$ is known and is generally ill-conditioned. $\mathcal{A}$ typically performs a Fourier transform (FT) on an irregular set, or a Radon transform, *etc.* $\boldsymbol{n}$ is the observation noise, which is supposed white, Gaussian, with known variance σ^2.

The reconstruction of $\boldsymbol{x}^*$ from $\boldsymbol{y}$ is an *ill-posed* inverse problem [2]; the solution $\hat{\boldsymbol{x}}$ is defined as the maximum of the posterior law $P(\boldsymbol{x}|\boldsymbol{y})$, or equivalently, the (global) minimum of the posterior energy $\mathcal{E}$:

$$\hat{\boldsymbol{x}} = \arg\min \mathcal{E}(\boldsymbol{x}) \tag{2}$$

$$\mathcal{E}(\boldsymbol{x}) = ||\mathcal{A}\boldsymbol{x} - \boldsymbol{y}||^2 + \Phi(\boldsymbol{x}); \tag{3}$$

where Φ is parameterised by (α, λ) and corresponds to the log-prior.

The edges of a locally homogeneous signal contain a crucial visual information. They can be addressed using a sum of truncated quadratic potential functions:

$$\Phi(\boldsymbol{x}) = \sum_{i,j\in\mathcal{V}_i} \phi(x_i - x_j), \tag{4}$$

$$\phi(t) = \begin{cases} (\lambda t)^2 & \text{if} \quad |t| < T \\ \alpha & \text{if} \quad |t| \geq T \end{cases}, \quad T = \frac{\sqrt{\alpha}}{\lambda}, \tag{5}$$

where $\mathcal{V}_i$ stands for neighbourhood of i and t is a first-order difference – a transition, $t_{i,j} = x_i - x_j, j \in \mathcal{V}_i$. Thus defined, Φ models a *piecewise Gaussian Markov random field with an implicit boolean edge process*[1].

Indeed, for $|t| < T$ the transition t is regularised with $(\lambda t)^2$, and i, j belong to the same homogeneous zone, while for $|t| \geq T$, $\phi(t) = const$, so that an edge is located between i and j. Then T is the *prior discontinuity detection threshold.*

2. The model parameters

$\mathcal{E}$ has numerous local minima: they represent various alternatives for the edge process. It is the model parameters (α, λ) which control whether the global minimiser corresponds to the most reliable edge process.

Parameter selection remains an open question in estimation theory. Different approaches can be devised. The empirical one considers (α, λ) as regulation "knobs". Statistics provide a frame for their estimation from the data or from some priors; this often leads to considerable numerical difficulties (*e.g.* [3]).

A third approach, qualifiable as *analytical,* aims to "correctly" position $\hat{\boldsymbol{x}}$ – the minimum of $\mathcal{E}$ – by the means of (α, λ). (Note that this is rarely feasable.) In our context, we shall select (α, λ) such that for some classes of original signals $\boldsymbol{x}^*$, the estimate $\hat{\boldsymbol{x}}$ has the same edges as $\boldsymbol{x}^*$, and is the closest to $\boldsymbol{x}^*$.

[1] Let $\mathbf{1} = [1, \ldots, 1]^T$; we suppose that $\mathbf{1} \notin \ker \mathcal{A}$, else $\forall \boldsymbol{x} \in \mathbb{R}^M$, $\forall k \in \mathbb{R}$, $\mathcal{E}(\boldsymbol{x} + k\mathbf{1}) = \mathcal{E}(\boldsymbol{x})$. Then $\mathcal{E}$ is integrable, while Φ is integrable only for $\sum x_i$ fixed; in our case, this has no practical consequences.

0-7803-2431-5/95 $4.00 © 1995 IEEE

For a *direct* observation $\mathcal{A} = I$, the behaviour of $\hat{x}$ as a function of (α, λ) has been analytically studied in [4], [1], [5], *etc.* This has given rise to several *posterior edge detection thresholds* of x^*, dependant on (α, λ).

For a general observation operator $\mathcal{A} \neq I$, these thresholds *cannot* be calculated analytically any longer; instead, they can be evaluated numerically. They permit in particular to quantify the *resolution* of the reconstruction – the finest features, which can be drawn by the edge process involved in $\hat{x}$. In our context, the resolution appears to be strongly limited by *both* the spectrum of $\mathcal{A}$ and the noise resistance requirement. Conversely, the optimal (α, λ) should incorporate a compromise between a desired resolution – the size of the smallest features sought for – and the resolution effectively allowed by $\mathcal{A}$ and the stability requirement.

The method proposed below works for operators, for which the contributions of the samples x_i^* to the data are *similar*. This requirement, physically appealing, is precised later; note that it fails for the Laplace transform, which needs a specific treatment [6].

3. Posterior thresholds

We consider several 1D 1-height original templates $x^*(1)$, which are simple enough but exhibit the edge detection capability of the estimator, and permit to determine in the noiseless case the set of the possible local minima of $\mathcal{E}$. These are mainly *steps* s, *gates* g and *ramps* r. Construct the family of original θ-height templates $x^*(\theta)$ – scaled copies of $x^*(1)$:

$$x^*(\theta) = \theta x^*(1). \qquad (6)$$

For $n = 0$ the "clean" data $y^* = y - n$ are also scaled:

$$y^*(\theta) = \mathcal{A}x^*(\theta) = \theta y^*(1). \qquad (7)$$

Let $x^*(\theta)$ have a θ-height jump located at $(m, m+1)$. Suppose that for $\theta \in \{\theta_0, \theta_1\}$, $\mathcal{E}$ admits two local minimisers – $z^{(0)}(\theta)$ continuous at $(m, m+1)$ while $z^{(1)}(\theta)$ with a jump at $(m, m+1)$ – and that it exists a Θ, such that for $\theta < \Theta$ the global minimiser is $\hat{x} = z^{(0)}(\theta)$ while for $\theta > \Theta$ it is $\hat{x} = z^{(1)}(\theta)$. We call this critical value Θ the *posterior edge detection threshold.*

Remark that each local minimiser is *linear* with respect to the data. Let D be the first-order difference operator: it is $(M - 1 \times M)$, circulant and its first row is $[-1, 1, 0, \ldots, 0]$. Let $k = k_1, \ldots, k_n$ and let $D_k = D_{k_1 \ldots k_n}$ be the same as D except the rows $k_1, .., k_n$, which are entirely zero. If $z^{(0)}(\theta)$ has edges at $(k_1, k_1+1), \ldots, (k_n, k_n+1)$,

$$\Phi(z^{(0)}(\theta)) = \lambda^2 \sum_{i \notin k} [z_{i+1}^{(0)}(\theta) - z_i^{(0)}(\theta)]^2 + n\alpha, \qquad (8)$$

so for $\theta_0 < \theta < \theta_1$, $z^{(0)}(\theta)$ is the minimiser of

$$\begin{aligned} \mathcal{F}_k(x) &= \|\mathcal{A}x - y^*(\theta)\|^2 + \|\lambda D_k x\|^2, & (9) \\ z^{(0)}(\theta) &= H_k y^*(\theta), & (10) \end{aligned}$$

where, in order to simplify the presentation, we used

$$\left[\begin{aligned} H_k &= (\mathcal{A}^T\mathcal{A} + \lambda^2 D_k^T D_k)^{-1}\mathcal{A}^T, \\ H &= (\mathcal{A}^T\mathcal{A} + \lambda^2 D^T D)^{-1}\mathcal{A}^T. \end{aligned} \right. \qquad (11)$$

(Note that if $k = \emptyset$, then $\mathcal{F} = \|.\|^2 + \|\lambda D x\|^2$.) Similarly, $z^{(1)}(\theta)$ is the minimiser of $\mathcal{F}_{k,m}$ and

$$z^{(1)}(\theta) = H_{k,m} y^*(\theta). \qquad (12)$$

The relevant posterior energies are

$$\left[\begin{aligned} \mathcal{E}(z^{(0)}(\theta)) &= \mathcal{F}_k(z^{(0)}(\theta)) + n\alpha, \\ \mathcal{E}(z^{(1)}(\theta)) &= \mathcal{F}_{k,m}(z^{(1)}(\theta)) + (n+1)\alpha, \end{aligned} \right. \qquad (13)$$

so that the global minimiser $\hat{x}$ is non-linear w.r.t. y:

$$\left[\begin{aligned} \mathcal{E}(z^{(0)}(\theta)) < \mathcal{E}(z^{(1)}(\theta)) &\mapsto \hat{x} = z^{(0)}(\theta) \\ \mathcal{E}(z^{(0)}(\theta)) \geq \mathcal{E}(z^{(1)}(\theta)) &\mapsto \hat{x} = z^{(1)}(\theta). \end{aligned} \right. \qquad (14)$$

The latter system exhibits the *piecewise continuity* of $\hat{x}$ w.r.t. y. Θ is the solution in θ of the equation

$$\mathcal{E}(z^{(0)}(\theta)) = \mathcal{E}(z^{(1)}(\theta)), \qquad (15)$$

which, using that $\mathcal{F}(x(\theta)) = \theta^2 \mathcal{F}(x(1))$, leads to

$$\Theta = \sqrt{\frac{\alpha}{\mathcal{F}_k(z^{(0)}(1)) - \mathcal{F}_{k,m}(z^{(1)}(1))}}. \qquad (16)$$

Clearly, Θ accounts for $\mathcal{A}$ and depends on (α, λ).

The step recovery

Consider the original signal (Fig. 1(a))

$$x^*(\theta) = \theta s_m, \quad s_{mi} = 0\,\mathbb{I}_{i \leq m} + \mathbb{I}_{i > m}, \quad \theta \in \mathbb{R}, \qquad (17)$$

where $\mathbb{I}$ stands for indicator function and s_m is a Heaviside function centered at $(m, m+1)$ with elements s_{mi}.

Given the noise-free observation $y^*(\theta)$, $\mathcal{E}$ can have at most two local minimisers:

$$z_s^{(0)}(\theta) = \theta H \mathcal{A} s_m \quad \text{and} \quad z_s^{(1)}(\theta) = \theta s_m. \qquad (18)$$

Indeed, $\mathcal{E}$ can have an unique continuous local minimiser. When $\mathcal{E}$ has a local minimiser with an edge, the latter can only be located at $(m, m+1)$, and $\mathcal{E}(z^{(1)}(\theta)) = \alpha$, because otherwise $\mathcal{E} > \alpha$.

Finally, the posterior threshold Θ_{s_m} reads

$$\Theta_{s_m} = \sqrt{\frac{\alpha}{\mathcal{F}(z_s^{(0)}(1))}}. \qquad (19)$$

Remark that $\mathcal{F}$ does not depend on α. The decrease of α favours the edge creation; conversely, $\alpha = \infty$ leads to the regularised least-squares.

The gate recovery

Consider now an a-width gate (Fig. 1(b)):

$$\boldsymbol{x}^*(\theta) = \theta \boldsymbol{g}_m(a), \quad \boldsymbol{g}_m(a) = \boldsymbol{s}_m - \boldsymbol{s}_{m+a}. \tag{20}$$

Let $\Theta_{\boldsymbol{g}_m}(a)$ be the threshold for the recovery of both edges. Often, it is reasonable to assume that they appear simultaneously in the global minimiser[2]:

$$\Theta_{\boldsymbol{g}_m}(a) = \sqrt{\frac{2\alpha}{\mathcal{F}(\boldsymbol{z}_{\boldsymbol{g}}^{(0)}(1))}}, \tag{21}$$

$$\boldsymbol{z}_{\boldsymbol{g}}^{(0)}(1) = H\mathcal{A}\boldsymbol{g}_m(a). \tag{22}$$

When a decreases, $\Theta_{\boldsymbol{g}_m}(a)$ increases (see Fig. 2(a)), and for a finite, $\Theta_{\boldsymbol{g}_m}(a) > \Theta_{\boldsymbol{s}_m}$.

The ramp breaking

Introduce the b-width ramp $\boldsymbol{r}_m(b)$ and consider $\boldsymbol{x}^*(\theta)$

$$\boldsymbol{x}^*(\theta) = \theta \boldsymbol{r}_m(b), \tag{23}$$

$$r_{mi}(b) = 0\,\mathbb{1}_{i \le m} + \frac{i-m}{b}\mathbb{1}_{m<i<m+b} + \mathbb{1}_{i \ge m+b}. \tag{24}$$

The gradient of $\boldsymbol{x}^*$ for $m < i < m+b$ is $\gamma = \theta/b$. When it is large, the ramp breaks at some p which is close to the middle (p could be determined numerically). Then the two alternative solutions are

$$\boldsymbol{z}_{\boldsymbol{r}}^{(0)}(1) = H\mathcal{A}\boldsymbol{r}_m(b) \quad \text{and} \quad \boldsymbol{z}_{\boldsymbol{r}}^{(1)}(1) = H_p\mathcal{A}\boldsymbol{r}_m(b), \tag{25}$$

so an erroneous edge apears for

$$\Theta_{\boldsymbol{r}_m}(b) = \sqrt{\frac{\alpha}{\mathcal{F}(\boldsymbol{z}_{\boldsymbol{r}}^{(0)}(1)) - \mathcal{F}_p(\boldsymbol{z}_{\boldsymbol{r}}^{(1)}(1))}}. \tag{26}$$

As previously, $\Theta_{\boldsymbol{r}_m}(b)$ increases when b decreases. If $\theta \gg \Theta_{\boldsymbol{r}_m}$, several edges can appear in the solution.

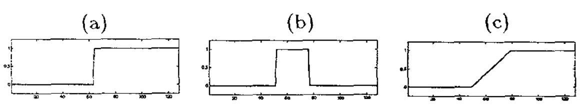

Fig. 1. Original templates. (a) Step. (b) Gate (c) Ramp.

4. The role of $\mathcal{A}$

The samples of $\boldsymbol{x}^*$ can be said "equitably" represented in the data if, far from the boundaries (at a minimum distance b), *i.e.* for $\forall m$, $m \in [b, M-b+1]$, the step detection threshold is *position independent* $\Theta_{\boldsymbol{s}_m} = \Theta_{\boldsymbol{s}}$. From Eq.(19) this occurs if

$$\sum_{i=m}^{M}\sum_{j=m}^{M} L_{i,j} = \sum_{i=1}^{M-m}\sum_{j=1}^{M-m} L_{i,j} = const, \quad \forall m \in [.], \tag{27}$$

$$L = \mathcal{A}^T(I - \mathcal{A}H)\mathcal{A}. \tag{28}$$

[2]Otherwise, 4 possible local minimisers must be compared: continuous, with an edge at $(m, m+1)$, with an edge at $(m+a, m+a+1)$, with 2 edges.

This relation is satisfied for shift-invariant convolutions, operators derived from the FT... In the sequel, we suppose it satisfied[3], and it turns out that we can also use $\Theta_{\boldsymbol{g}}(a)$ and $\Theta_{\boldsymbol{r}}(b)$ with $p = m + \mathrm{int}(b/2)$.

The resolution of the reconstruction (even in the noise-free case) is limited by the spectrum of $\mathcal{A}$. Let $\mathcal{A}$ perform an incomplete FT, filtered over a rectangular window $[-F, F]$; the decrease of $\Theta_{\boldsymbol{g}}(a = 8)$ when the cut-off frequency F increases, is given in Fig. 2(b). For small Fs, a ramp breaks easily – Fig. 2(c).

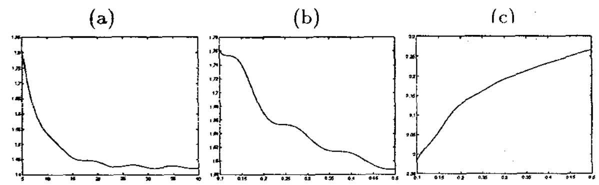

Fig. 2. Posterior edge detection thresholds. $\mathcal{A}$ performs an incomplete FT on $[-F, F]$ (reduced frequencies); $(\alpha, \lambda) = (5, 5)$. **(a)** Gate recovery as a function of a: $\Theta_{\boldsymbol{g}}(a)$, $a = 5, .., 40$, $F = 0.125$. **(b)** Gate recovery as a function of F: $\Theta_{\boldsymbol{g}}(8)$ for $F = [0.1, .., 0.5]$. **(c)** Ramp break: $\Theta_{\boldsymbol{g}}(20)$ for $F = [0.1, .., 0.5]$.

5. Accounting for the noise

Since the estimator is piecewise continuous, it is *impossible* to guarantee that in the presence of noise, $\hat{\boldsymbol{x}}$ always remains in the vicinity of the noise-free solution. Instead, following [1], it can be required that in the presence of noise, the solution corresponding to a zero-valued original object, is everywhere continuous.

Let $\boldsymbol{x}^* = \boldsymbol{0}$, then $\boldsymbol{y} = \boldsymbol{n}$. The continuous minimiser

$$\boldsymbol{z}_{\boldsymbol{n}}^{(0)} = H\boldsymbol{n} \tag{30}$$

is global provided that

$$|DH\boldsymbol{n}| < \Theta_{\boldsymbol{s}}\boldsymbol{1}. \tag{31}$$

The latter condition is strong, since the noise is white and hence can give rise to jumps which are close together ($a = 1, 2$) rather than isolated, and then $\Theta_{\boldsymbol{g}}(a) \gg \Theta_{\boldsymbol{s}}$. Since $Pr(|n_i| < 2\sigma) \approx 0.95$, in practice it is sufficient to require

$$2\sigma \max\{|DH|\boldsymbol{1}\} < \Theta_{\boldsymbol{s}}, \tag{32}$$

where $\max\{.\}$ stands for maximal element.

On the other hand, inside a homogeneous region, a transition is regularised with $(\lambda t)^2$; locally, λ can be regarded as $\lambda = \sigma/\sigma_t$, where σ_t is the variance of t. So, large λs create weakly varying homogeneous regions.

[3]Otherwise, we could set:

$$\Theta_{\boldsymbol{s}} = \min_m \Theta_{\boldsymbol{s}_m}, \quad \Theta_{\boldsymbol{g}} = \max_m \Theta_{\boldsymbol{g}_m}, \quad \Theta_{\boldsymbol{r}} = \min_m \Theta_{\boldsymbol{r}_m}. \tag{29}$$

6. A set of constraints

The priors on $\boldsymbol{x}$, needed for the selection of (α, λ) are

- $(a_{\min}, \theta_{\min})$: the minimum width of a homogeneous region and the minimum height between two regions;

- $(b_{\max}, \gamma_{\max})$: the maximum length and gradient of a ramp inside a homogeneous region.

Since $\Theta_{\boldsymbol{g}}(a) < \Theta_{\boldsymbol{g}}(a_{\min})$ if $a > a_{\min}$, and $\Theta_{\boldsymbol{r}}(b) > \Theta_{\boldsymbol{r}}(b_{\max})$ if $b < b_{\max}$, these priors yield

$$\Theta_{\boldsymbol{g}}(a_{\min}) < \theta_{\min} \quad \text{and} \quad \Theta_{\boldsymbol{r}}(b_{\max}) > b_{\max}\gamma_{\max}. \qquad (33)$$

Finally, the *admissible* (α, λ) belong to the intersection of

$$\alpha < s_1(\lambda), \quad \alpha > s_2(\lambda), \quad \alpha > s_3(\lambda), \qquad (34)$$

where s_1, s_2, s_3 do not depend on α and can be evaluated for some range of variation of λ:

$$\begin{aligned} s_1(\lambda) &= \theta_{\min}^2 \frac{1}{2} \mathcal{F}(\boldsymbol{z}_{\boldsymbol{g}}^{(0)}(1)), \\ s_2(\lambda) &= \gamma_{\max}^2 b_{\max}^2 [\mathcal{F}(\boldsymbol{z}_{\boldsymbol{r}}^{(0)}(1)) - \mathcal{F}_p(\boldsymbol{z}_{\boldsymbol{r}}^{(1)}(1))], \\ s_3(\lambda) &= \sigma^2 4 (\max\{|DH|\mathbf{1}\})^2 \mathcal{F}(\boldsymbol{z}_{\boldsymbol{s}}^{(0)}(1)). \end{aligned}$$

In Fig. 3(a), such an admissible set is shown. The resolution limitations are quite evident: *e.g.* a small decrease of $\theta_{\min}$ will make this set empty. When this occurs, a compromise is imperative.

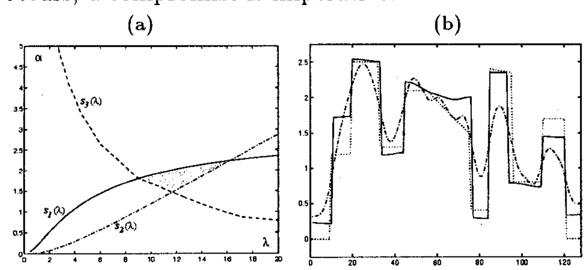

Fig. 3. Parameter selection and reconstruction. $\mathcal{A}$ performs an FT, filtered using $w(n) = \exp(-60(n - \frac{M}{2})^2)$, noisy data with $\sigma = 0.6$. **(a)** The admissible set (the shaded area) $s_i(\lambda) = \alpha$ versus λ, given $\sigma = 0.6$, $(a_{\min}, \theta_{\min}) = (8,1)$, $(b_{\max}, \gamma_{\max}) = (20, 0.03)$; $s_1(\lambda)$ (-), $s_2(\lambda)$ (-.-), $s_3(\lambda)$ (- -). **(b)** Reconstructions of the original signal (..): Usual Gaussian MM with with $(\alpha, \lambda) = (\infty, 3)$(-.), PG MM with $(\alpha, \lambda) = (1.6, 11)$ (-).

7. Extension to images

A similar approach can be adopted for the determination of the parameters for the reconstruction of images. In this case, the original templates are *2D steps, square patches* and *2D ramps.* Here again, a continuous local minimiser $\boldsymbol{z}^{(0)}$ and a local minimiser with edges $\boldsymbol{z}^{(1)}$ are compared.

A condition for the representation of the pixels in the data, equivalent to Eq.(28), is straightforward; we suppose that it is satisfied.

Let $\boldsymbol{x}^*$ be an image with edges – a square patch or a step. Even when its pixels are "equitably represented" in $\boldsymbol{y}$, the edges of $\boldsymbol{z}^{(1)}$ can be slightly different from the edges of $\boldsymbol{x}^*$: these differences can occur at the corners and near to the boundaries, but they exist only for a tiny range of variation of α. It is then a numerically negligible approximation to take $\boldsymbol{z}^{(1)} = \boldsymbol{x}^*$.

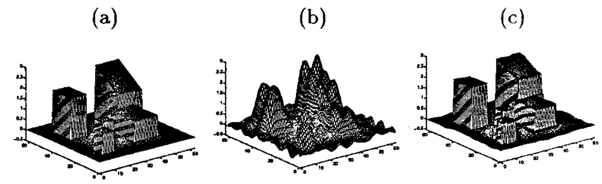

Fig. 4. Fourier synthesis example. (a) Original image. The data is the incomplete FT on [−0.125,0.125] with $\sigma = 0.63$. **Reconstructions: (b)** Usual Gaussian MRF prior. **(c)** PG MM: $(\alpha, \lambda) = (0.4, 16)$ assures a threshold for the 5×5 patch equal to 1.3 and does not break gradual transitions up to 0.02 per pixel. $\mathcal{E}$ is minimised using a GNC algorithm [7].

8. Concluding remarks

We propose a technique for the numerical determination of (α, λ), aimed at ensuring a correct edge recovering. It is *optimal* in the sense that it provides the maximum resolution allowed by the noise statistic and by the observation operator. (α, λ) should be determined once for a given class of reconstruction problems.

Its extension to some other functions, such as $\phi(t) = 1 - d(t)$, where d is the Kronecker, is straightforward.

9. References

[1] A. Blake and A. Zisserman, *Visual reconstruction.* Cambridge: The MIT Press, 1987.

[2] G. Demoment, "Image reconstruction and restoration : Overview of common estimation structure and problems," *IEEE Transactions on Acoustics Speech and Signal Processing*, vol. 37, pp. 2024–2036, Dec. 1989.

[3] H. Derin and S. Güler, "Realizations and parameter estimation for line processes," in *Proceedings of IEEE ICASSP*, pp. 2213–2216, 1990.

[4] D. Mumford and J. Shah, "Boundary detection by minimizing functionals," in *Proceedings of IEEE ICASSP*, pp. 22–26, 1985.

[5] A. Chambolle, "Dualité mumford-shah/canny-deriche et segmentation progressive d'images," in *GRETSI*, pp. 667–669, 1993.

[6] M. Nikolova and A. Mohammad-Djafari, "Discontinuity reconstruction from linear attenuating operators using the weak-string model," in *Proceedings of European Signal Processing. Conf.*, vol. 2, pp. 1062–1066, 1994.

[7] M. Nikolova, A. Mohammad-Djafari, and J. Idier, "Inversion of large-support ill-conditioned linear operators using a markov model with a line process," in *Proceedings of IEEE ICASSP*, vol. 5, pp. 357–360, 1994.