Topics in Applied Maths II: **Calculus of Variations** MAT6112 - CUHK 2008

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I. BASIS OF THE THEORY WITH EXAMPLES

1) Functionals
2) Function spaces
3) Variation of a functional
4) Extrema
5) Euler-Lagrange (E-L) equation
6) The case of several variables
7) Variational derivative
8) Invariance of E-L equation

II. GENERALIZATIONS

1) Fixed end point problem
2) Parametric form
3) Functionals involving higher-order derivatives
4) Illustration of modeling
5) Constrained problems
6) Finite subsidiary constraints
7) General variation of a functional
8) Hamiltonian
9) End points on two given curves or surfaces
10) Broken extremals

III. DUAL SPACES

1) Linear functionals
2) Examples
3) Extension of linear functionals
4) The second dual space
5) Minimum-norm problems
6) Applications
7) Weak convergence (reminder)
8) Hyperplanes and convex sets
9) The Minkowsky functional and separation theorems
10) Duality in minimum norm problems

IV. CONVEX SETS AND FUNCTIONALS

1) Main properties
2) Epigraphs
3) Convex (concave) conjugate functionals
4) Dual optimization (Fenchel duality theorem)
5) Examples and applications
6) Min-Max theorems of Game Theory

V. DIRECT METHODS

1) Topologies
2) Weak compactness
3) Existence of minimizers
4) Bad examples
5) Relaxation

SOME REFERENCES

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